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Super-resolved imaging with randomly distributed, time- and size-varied particles

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Abstract
In this paper we present a super-resolved approach aimed at overcoming the diffraction limit in imaging systems. It is based on place randomly and time-varied particles having different sizes on the top of the sample. By considering particle sizes smaller than the object's minimum detail that an imaging system can resolve, it is possible to recover a high resolution image from a set of low resolution images while before capturing each image we produce a randomly modified distribution of the particles by vibrating the sample. The simulation process as well as experimental results validates the proposed approach that includes effectively decreasing the F number of the imaging system while being capable of allowing super-resolved imaging.

Keywords: spatial resolution, optical diffraction, imaging and optical processing

1. Introduction

The resolution of an optical imaging system can be defined as the smallest spatial separation distance between two features that can still be resolved by the system. According to theory, this minimal separation distance is proportional to the optical wavelength and the F number of the imaging lens [1].

The various approaches for overcoming those limitations are called super-resolving techniques. In order to exceed the resolving limitations of an imaging system one needs to convert the spatial degrees of freedom into other domains (encoding), to transmit them through the optical system and then to reconstruct the image (decoding) [2–4]. The domains that may be used in order to multiplex the spatial information are the time [5–8], polarization [9, 10], wavelength [11, 12], field of view or spatial dimensions [13–16], code [17–19] and gray level [20].

Time-multiplexing is one of the most applicable techniques for super-resolution. The basic principle included the use of two moving gratings [5–8] where the first encodes the spatial information and is projected or attached to the object [21] and the second, which is responsible for the decoding of the information, is placed near the detector and may be realized digitally by capturing a set of images and multiplying each image in the set by the proper distribution [22]. The two gratings must move or change from one image in the sequence to the other. This relative movement or change allows the proper decoding of the encoded information.

Later on, instead of a periodic pattern a random pattern of speckles was used to improve the resolution [23–26]. This random pattern encoded the high spatial frequencies existing in the object. After time-multiplexing and by knowing the encoding pattern one may decode the information and construct the high resolving image of the object. However, the main drawback of that approach is that the high resolution random decoding pattern had to be *a priori* known.

A common point of all the super-resolving approaches is that the masks used in the encoding should have a critical size below the diffraction limit of the imaging system. Then, it is possible to shift the high spatial frequency content of the object towards the limited aperture of the imaging system. This spectral shift allows the transmission of such a high frequency band through the system aperture and can be recovered and replaced using the proper decoding. But in any case, the encoding mask must diffract such high-order components allowing them to cross through the system aperture. This fact means that the encoding mask should have a structure smaller than the minimal details the optical system can image.

In this paper we propose a super-resolved approach that is related to general imaging systems. Similar to what was demonstrated in [27] where the authors used rain droplets as the encoding super-resolving pattern, we now use a distribution of random particles with different sizes and varying in time. Such a random distribution is used as the encoding mask and it has the particularity that the particles are a sparse distribution, allowing us to extract the high resolution decoding pattern from the low resolution images. Since the pattern of each particle is blurred because the particle sizes are smaller than the imaging system resolution and the blurring spots are spatially separated due to the sparse distribution, their position may be digitally extracted. This fact means that we are extracting the decoding pattern without any *a priori* assumption regarding the knowledge of the encoding one. This is an important fact in far range imaging because the medium between the object and the imaging system modifies this pattern, for instance due to turbulence.

The extraction of the decoding pattern is performed by digital processing as follows. First a low resolution image of the target without the particles is captured. Then we capture a set of low resolution images with the particles. From each one of those images, we subtract the low resolution image without the particles and obtain the low resolution image of the randomly distributed particles (this image is changed for each image in our sequence). This random low resolution image is sparse and thus we may allocate the centers of the blurred spots corresponding to the particles' positions. Then we construct a high resolution image with small spots corresponding to the position of the particles in that specific image and use it as the decoding image, i.e. we multiply this reconstructed high resolution decoding pattern with the low resolution image with the particles. We sum all the decoded images, subtract out of it the original low resolution image and obtain the final high resolution reconstruction.

This paper is organized as follows. The theoretical background is presented in section 2. The simulation process and experimental validation may be seen in section 3. The paper is concluded in section 4.

2. Theoretical background

The mathematical proof of principle is similar to the derivation seen in [27] or in [28]. In the following, we give some explanations about the methodology used in the proposed approach. Once we capture a low resolution image of the object without the particles, we add the particles and capture a set of low resolution images. From each image in the set, we reconstruct the random distribution of the particles using numerical processing. This is done by subtracting the low resolution image of the object (so that we are left with the low resolution distribution of the particles) and allocating the centers of the blurring spots. Those centers are our high resolution decoding pattern. The extraction of the high resolution decoding pattern from low resolution images is possible since the random distribution of the particles is sparse.

After extracting the decoding pattern of each image in the sequence, we multiply and sum all the images. We subtract from the summation the low resolution image of the object and obtain the high resolution reconstruction. Before capturing each image in the sequence we slightly touch the object which causes a change in the random and sparse distribution of the particles. If we denote by $s(x)$ the high resolution distribution of the object, by $p(x)$ the blurring point spread function (PSF) of our imaging camera and by $f(x,t)$ the time-varying random and sparse pattern (x and t are the spatial and the temporal coordinates, respectively), each low resolution image in the sequence that is captured by the camera equals

$$\int_{x'} s(x')f(x',t)p(x-x')\,\mathrm{d}x'. \tag{1}$$

The high resolution decoding pattern is digitally extracted following the numerical procedure previously explained. The reconstruction $r(x)$ is

$$r(x) = \int_t \left[\int_{x'} s(x')f(x',t)p(x-x')\,\mathrm{d}x'\right]\tilde{f}(x,t)\,\mathrm{d}t \tag{2}$$

where $\tilde{f}(x,t)$ is the digitally estimated decoding pattern. Due to the random distribution of the encoding/decoding pattern we assume

$$\int_t f(x',t)\tilde{f}(x,t)\,\mathrm{d}t = \delta(x'-x)+\kappa \tag{3}$$

where κ is a constant. The expression presented in equation (3) is an approximated model. In order to show how good this modeling is we performed the simulations appearing in figure 1. To obtain the results of figure 1 we took 100 random vectors with normal distribution (x and x' as well as the time axis; all three have 100 components). After performing the multiplication and averaging over the time axis as depicted in equation (3) we obtained the result of figure 1(a). In this simulation the encoding/decoding patterns were identical. In order to test how variations affect the obtained result, we added a random normally distributed noise to the decoding pattern while the noise has a standard deviation of 3 (three times larger than the standard deviation of the random pattern itself). The results are presented in figure 1(b). The diagonal line now has lower contrast but it is still very visible. The obtained diagonal line of figure 1 designates that, indeed, approximating the left-hand side of equation (3) to $\delta(x'-x)$ is a sufficiently good assumption.

By changing the order in equation (2) and using equation (3) we obtain

$$\begin{aligned} r(x) &= \int_{x'} s(x')\left[\int_t f(x',t)\tilde{f}(x,t)\,\mathrm{d}t\right]p(x-x')\,\mathrm{d}x' \\ &= \int_{x'} s(x')\delta(x'-x)p(x-x')\,\mathrm{d}x' \\ &\quad + \kappa\int_{x'} s(x')p(x-x')\,\mathrm{d}x' \\ &= s(x)p(0) + \kappa\int_{x'} s(x')p(x-x')\,\mathrm{d}x'. \end{aligned} \tag{4}$$

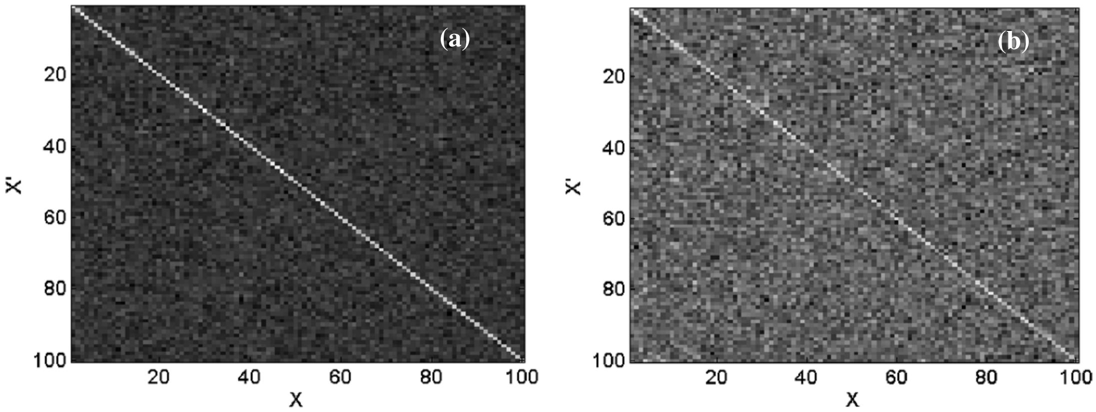

Figure 1. Numerical demonstration for equation (3). (a) The encoding/decoding patterns are identical. (b) The decoding pattern is equal to the encoding but with the addition of normal noise having three times larger standard deviation.

What we see in equation (4) is that $r(x)$ equals the high resolution object $s(x)$ multiplied by a constant $p(0)$ and summed with its low resolution version. Thus, by subtracting from the reconstruction the low resolution version allows us to obtain the high contrast and high resolution reconstruction.

Loosely speaking, by placing the random high resolution pattern (sparse distribution of particles) attached to the object allows us to shift the high spatial frequencies into the low band range. Such high spatial frequencies are responsible for the smallest details of the object and, without the high resolution mask, are trimmed by the aperture of the imaging lens. Thus, changing the random distribution allows separation between the high frequencies that were folded into the low band pass and the originally low spatial frequency content. By multiplying each image in the stored sequence by a decoding pattern that is similar to the encoding one, a proper positioning of the folded spectral bands back to their original spectral location is accomplished.

Note that there are several parameters affecting the quality of the reconstruction. Such parameters are the sparseness of the particles, the number of captured frames and the quality of the estimated decoding pattern. The sparseness of the decoding pattern is related to the number of coded images. The point is that, since in every image the position of the particles is varied, if they are too sparse more frames are needed in order to cover the full field of view. Thus, the optimal situation is when the particles are separated by a distance equal to the PSF of the given imaging system and in this case the super-resolution principle is valid (we can still estimate the correct decoding pattern) and a lower number of frames should be captured (the number of frames in this case should be equal to the square of the super-resolving factor (because the super-resolution is performed in 2D) which is also the square of the ratio between the PSF and the particle's diameter). Also, if the particles are sparser over the same field of view, it means that there are less particles and thus the encoding/decoding patterns are less random which means that the condition of equations (3) is less valid.

The effect of a badly estimated decoding pattern is in the reduction of the high resolution portion in the reconstructed image. Therefore what is important there is the number of bits designated for the reconstruction computation. In the simulation of figure 1(b) we showed how additive noise added to the decoding pattern reduces the contrast of $\delta(x' - x)$ which means also the reduction of the energetic portion of the high resolution reconstruction in the decoded output distribution. Note also that increasing the number of captured frames makes the reconstructed image less noisy since it will become more averaged, more smoothened, more uniform and less dependent on the decoding pattern.

3. Validation of the approach

In order to validate the proposed method, we include both numerical simulations and experimental results. In the numerical investigation, we present the images obtained while using a resolution test chart considering two cases in order to show that a resolution improvement is possible in different cases by only selecting the size of the particles that are going to be used. This is an important fact because future efforts will be pursued in order to apply the proposed method to microscopy. Note that in microscopy in some cases the imaged object has particle-like morphology. In this case it is more difficult to have a proper estimation of the decoding pattern (one needs to have a proper estimation, otherwise the super-resolved reconstruction will not work). This difficulty can be simplified if one has some *a priori* information on where the particles are located.

In figure 2(a) we present a low resolution version of a resolution test target after applying blurring with a PSF of 5 pixel × 5 pixel. After applying the proposed approach while summing 500 images in the reconstruction process and using particles ranging from 1 to 2 pixels yields the image of figure 2(b). In figure 2(c) we present the low resolution image obtained with a blurring kernel (PSF) of 15 pixel × 15 pixel. A random pattern with particles of dimensions 1 to 2 pixels was used to improve the resolution. After summing 100 images in the sequence and following the above described procedure, it yields the result of figure 2(d). One may see the significant improvement obtained both in figure 2(b) as well as in figure 2(d).

Note that the smallest feature to be obtained in the reconstructed image depends only on the size of the particles

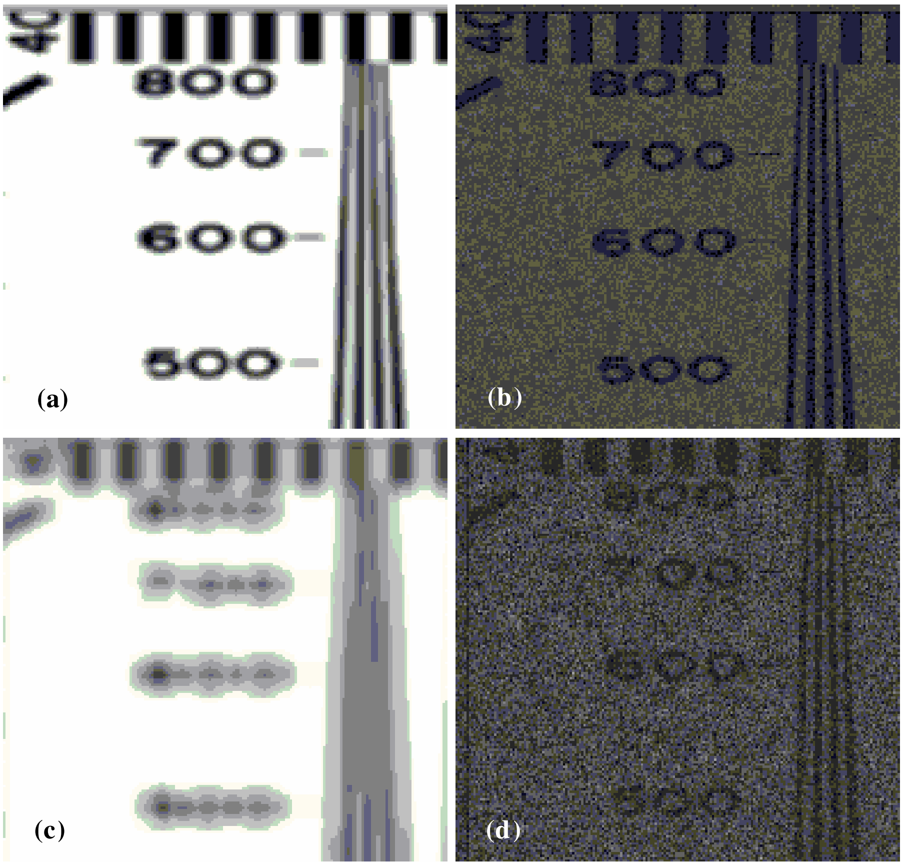

Figure 2. Numerical simulations.

and not the blurring kernel or the number of captured frames. Increasing the blurring PSF requires more frames only since in that case the particles should be more sparse and thus, to cover the full field of view, more frames are required. But the resolution of the final outcome should be the same. In the simulations of figures 2(b) and (d) the bandwidth of the reconstructed target is approximately the same since the size of the particles was the same. The image of figure 2(d) is more noisy because less frames were taken and thus the field of view was not as uniformly covered as in figure 2(b) and the difference between the estimated decoding pattern and the original encoding distribution of the particles was more dominant in the reconstruction. Nevertheless the 100 frames that were applied to obtain figure 2(d) were enough to reconstruct the spatial content of the input resolution test target.

The next step was to perform an experimental validation. The diagram depicted in figure 3 shows the whole process of the proposed approach used in the experimental validation. In the experimental configuration, a set of particles were randomly thrown over a printed version of a USAF resolution test target with overall dimensions of about 30 cm. 300 images were captured in the whole sequence. In each image the random distribution of the particles was changed by touching the USAF target. The average diameter of the ball-like particles was 1 cm. The positions of the particles are unknown and they were decoded from the set of low resolution images.

As the imaging system, we used a Canon PowerShot A710 camera with an F number of 8, focal length 5.8–34.8 mm and exposure time of $1/60$. The distance between the object and the imaging system is 1.5 m.

In figure 4(a) we present the low resolution image of only the particles' distribution. In figure 4(b) we show one low resolution USAF image out of the sequence captured by the camera (with particles). In figure 4(c) we present the low resolution image captured by the camera before adding the particles. This image is important as part of the numerical process for extraction of the decoding pattern. In figure 4(d) we present the super-resolved reconstruction. One may easily see the resolution improvement obtained in figure 4(d) in comparison to figure 4(c).

The artifacts obtained in the reconstruction of figure 4(d) depend on the number of frames taken in the averaging process. The more frames we take the more uniform and more averaged the reconstruction becomes. Since in this experiment the number of frames was not very large some artifacts related to the particles still remained.

4. Conclusions

In this paper we have presented an approach allowing significant resolution improvement of any imaging system. The basic novelty of the proposed approach is that super-resolved imaging may be obtained without having any *a priori* knowledge on the encoding pattern except the fact that it is a sparse one. The configuration involves randomly placing

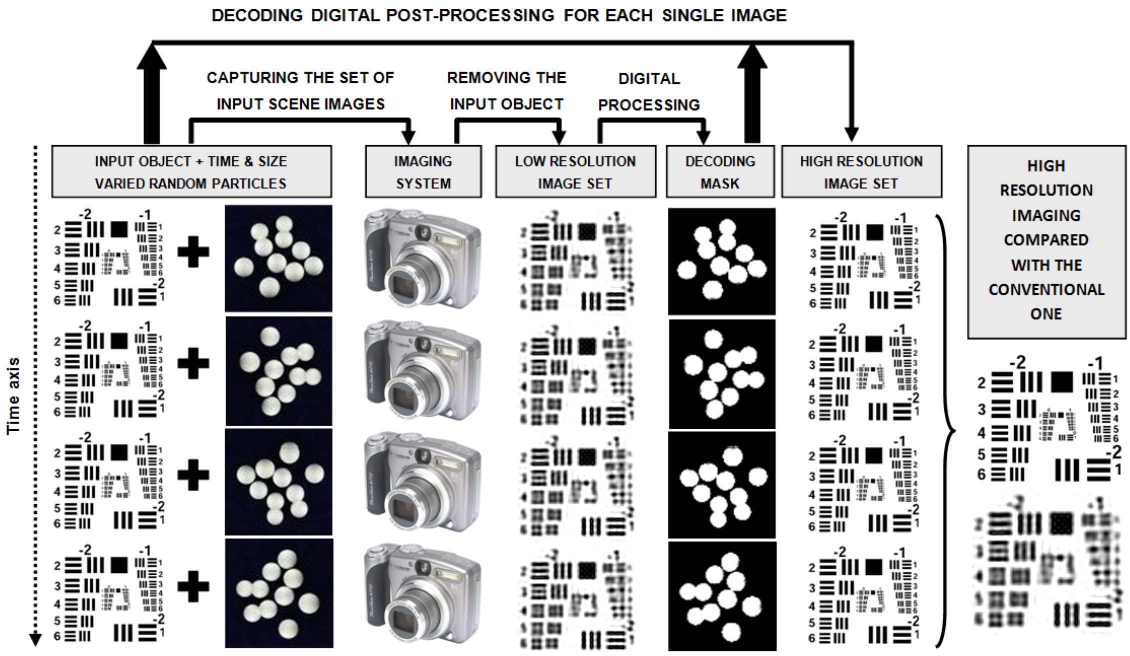

Figure 3. Schematic chart of the proposed methodology.

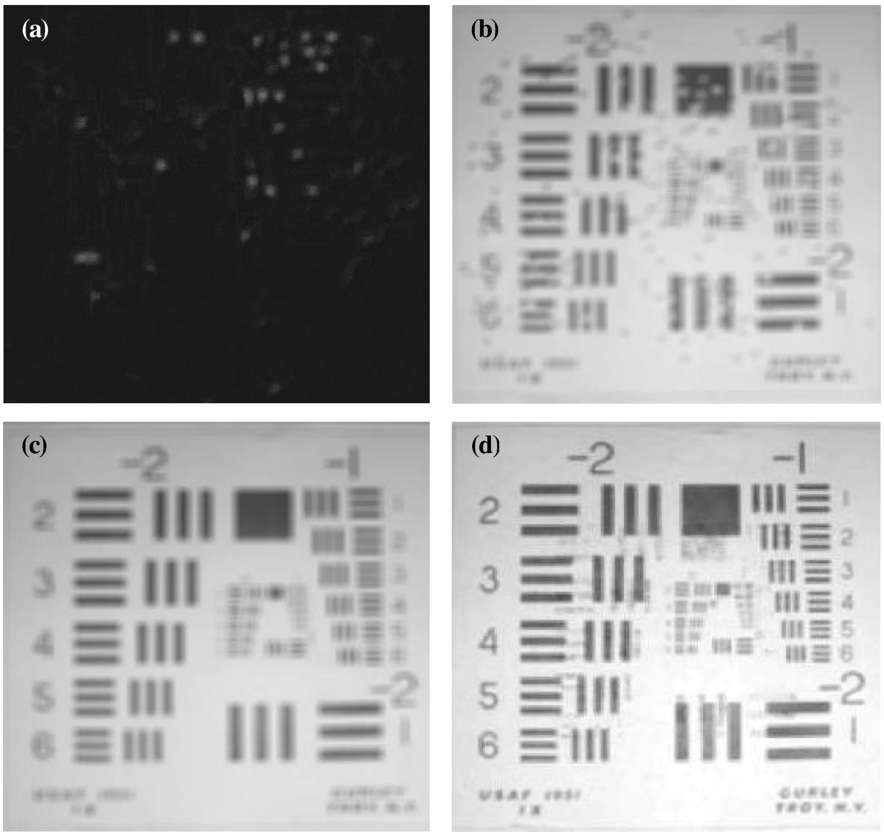

Figure 4. Experimental results. (a) Low resolution image of only the particles. (b) One low resolution USAF image out of the sequence captured by the camera (with particles). (c) Low resolution image captured by the camera (without particles). (d) The super-resolved reconstruction.

time- and size-varying particles on top of the object. A set of low resolution images are captured while before capturing each image the random distribution of the particles can be changed by touching the object or by moving the particles. Digital processing allows us to reconstruct a high resolution image out of the low resolution sequence. Numerical as well as experimental validation of the proposed approach was presented.

Note that, although in the experiment we have used a regular imaging system, in the future we aim to implement the proposed approach in microscopy in order to improve the diffraction resolution limit imposed by the objective lens. The improvement may yield an effective decrease in the *F* number as well as imaging of sub-wavelength features (in this case the particles should be positioned closer than one wavelength to the features, otherwise the sub-wavelength information carried by the evanescent waves is lost).

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